New Tools for Investment Decision-making: Real Options Analysis

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No consumer of the Canadian media could have failed to notice that one of North America’s largest utilities, Ontario Hydro, has been reeling financially over the last few years. The Globe and Mail recently reported “Ontario Hydro is taking a special one-time charge of $2.5 billion to write off surplus assets... [the reason being the utility] still faces major financial problems because of weaker-than-expected demand for electricity” (Globe and Mail, p. B3, Dec. 11, 1996). In large part Hydro’s problems are the result of over-expansion using large-scale inflexible nuclear power plants in anticipation of demand increases that did not materialize. The construction of these plants left Ontario Hydro with large amounts of debt and very high fixed operating costs, both of which limit its ability to compete in a more open electricity market. We argue in this article that traditional valuation methods may be to blame.

Standard valuation methods seem to be lacking in other contexts as well. The values of Netscape and Yahoo at their initial public offerings surprised many analysts because they seemed to defy the logic of traditional valuation models. For the price assigned to these companies by the market to be rational, it appeared that the sales and earnings of these companies would have to grow at remarkably high rates. Was this a case of market irrationality, or is there some other explanation?

Recent research has applied the model used to value stock options to value start-up companies and investment projects in general. Such research indicates that the high valuations assigned to Netscape and Yahoo may be rational, because such companies typically possess valuable sets of opportunities (or options). In contrast to the more familiar financial options (such as stock options), these opportunities are known as real options. The same research also indicates that a company, such as Ontario Hydro, that must periodically consider large capital projects, also is in possession of a series of options that should be incorporated into proper investment analysis. The failure to recognize their existence may lead to sub-optimal investments. In particular, using standard discounted cash flow (DCF) analysis may result in a bias in favor of large-scale expansion projects as opposed to a series of smaller incremental investments.

DCF analysis has been used for over 40 years and has become a well accepted part of the investment decision process. Successive surveys have shown the methodology gaining ground almost everywhere. As stated by Jog and Srivastava (1995), by 1991, "...the use of DCF methods has become a norm in Canadian firms..." While the large sample surveys detect an increased use of DCF techniques, they seldom asked which investments were subjected to this formal analysis. Although in theory any commitment of resources was amenable to analysis within the DCF framework, in practice companies almost never applied these techniques to R&D, advertising, or training, because such investments did not offer a simple cash flow-based pattern of return.

These shortcomings of DCF techniques have not escaped the critics [see, for example, Garvin (1982); Hayes and Garvin (1982); and Kaplan...
The objections to DCF include alleged conceptual weaknesses, inability to evaluate strategic investments with future growth opportunities, and especially bias against long-term projects. Michael and Millen (1984), for example, state that traditional financial valuation models are more suited to meet short-term profitability goals rather than long-term strategic goals. Bierman (1988) surveyed 102 chief financial officers on the problems of implementing capital-budgeting analysis. Difficulty in incorporating strategic considerations was the problem most frequently cited. Hayes and Garvin (1982) attacked the DCF methodology contending that it promoted "a policy of progressive disinvestment." They recommended suspending the strict financial logic embodied in DCF. Capital investment, they asserted, "represents an act of faith, a belief that the future will be as promising as the past, ...[and] a commitment to making that future happen." Nevertheless, DCF has staunch defenders: when these criticisms were initially levelled, they quickly drew a response which suggested that the problem was usually with the misapplication of DCF techniques rather than the techniques themselves [Myers (1984), Kulatilaka (1984)].

More recently, however, some capital budgeting analysts have recognized that some of the underlying assumptions to the traditional DCF approach, such as the Net Present Value (NPV) analysis, are routinely violated when it is implemented. In particular, implicit assumptions are made concerning an expected scenario of cash flows (based on management's commitment to a pre-determined path). For example, it is usually assumed that the project will continue to operate at a pre-determined base scale until the end of its pre-specified expected useful life (i.e. the investment is partially or completely irreversible). It also assumes an immediate initiation of the capital project (i.e. management has no leeway about the timing of the investment). In reality, however, as new information arrives and uncertainty about market conditions and future cash flows is gradually resolved, management is most likely to use its flexibility to alter its initial operating strategy in order to capitalize on favorable future opportunities or to mitigate potential losses.

As a rapidly growing literature has shown, management's ability to alter its future actions in response to changing market conditions can profoundly affect the decision to invest, and undermines simple DCF rules (see, among others, Majd and Pindyck (1987), Triantis and Hodder (1990), Ingersoll and Ross (1992), and Mauer and Triantis (1994)). The reason is that investment opportunities can be viewed as a collection of options on real assets (i.e. real options), which are analogous to financial call (or put) options. For example, a firm with a discretionary investment opportunity has the right – but not the obligation – to acquire the (gross) present value of expected cash flows by making an investment outlay on or before the anticipated date when the investment opportunity will cease to exist. As in financial options, the firm makes an irreversible investment expenditure (i.e. it exercises its option to invest) only when it is in its favour to do so. By exercising the option, the firm gives up the possibility of waiting for new information to arrive that might affect the desirability and/or timing of the expenditure. The fact is this lost option value is an opportunity cost that must be incorporated when the investment is analyzed.

Often a set of real options will be present. In such a case, a discretionary investment opportunity can be seen as a collection of real options (i.e. options on a portfolio consisting of the gross project value and other real call or put options). For example, the option to contract the scale of a project by Y per cent to save on certain planned maintenance expenditure of $X can be seen as a put option on Y per cent of the project's value with exercise price of $X. Thus, management may find it advantageous to build a plant with lower initial construction costs, but higher maintenance expenditures in order to acquire the flexibility to scale down the operation by reducing expenditures on maintenance if sales fall short of expectations. Alternatively, management may also have the flexibility to alter operation in any given year if revenues fall short of variable costs in that year. One may thus look at operating in each year as a call option on that year's cash revenue, the exercise price being the variable cost of operating. Further, a traditional DCF analysis will omit the opportunity to close a business if conditions merit. It will only include some expectation of negative cash flows. The option analysis, however, includes the appropriate response to bad conditions: close the busi-
ness and cut the losses. Thus it truncates bad outcomes, increasing value.

The purpose of this article is to show that the incorporation of real options analysis into capital budgeting decisions greatly enhances the ability of an organization to assess potential projects. It is argued that no technique previously available offers so much flexibility in addressing and valuing the numerous options that inevitably arise when one searches for a project in which to productively invest a firm’s capital. For this reason, the standard DCF method is likely to systematically undervalue investments with significant operating options.

We show that the use of real options analysis has the potential not only to conceptualize but also to quantify the value of options from active management and strategic interactions. We present three fairly simple examples that highlight three important real options. In the next section, the focus is on the option to defer investment. We use the example of a pharmaceutical company which must choose the optimal timing of an investment in a new drug. Section 3 considers the importance of input flexibility in the context of a mini-mill that has the choice of generating electricity using several potential energy sources. In the next section, we deal with the choice of the optimal scale of an investment. Here we consider a public utility that must expand capacity, but has discretion over the scale and speed of expansion. The final section points out other real options of potential interest, and considers several complications and extensions.

The option to defer investment

According to traditional DCF analysis, a project should be accepted only if the return on the project exceeds an appropriate hurdle rate. In the context of cash flows and discount rates, this translates into projects with positive net present value (NPV). As pointed out in the previous section, a major limitation of traditional investment analysis is that it fails to fully incorporate the myriad options that are usually associated with many projects.

Consider for example a pharmaceutical company that has been approached by an entrepreneur who has patented a new drug to treat ulcers. The entrepreneur has obtained the regulatory approval and has the patent rights. Although the drug shows promise, it is very expensive to manufacture and has a relatively small market. The situation is further complicated in that it is possible that another pharmaceutical company may enter the same market in the near future and the required investment is completely irreversible (i.e. the plant can only be used to make the ulcer drug). To keep matters simple, it is assumed that the factory can be built instantly at a cost of $3,200; the factory produces a constant quantity forever; the cost of production is zero; the quantity produced can be sold now for $400; and next year revenues will decrease to $200, if the other pharmaceutical company is successful in developing and getting approval for a similar drug, or, failing this eventuality, revenues will increase to $600. With probability $q = 0.5$, no competition enters, and the company can exert some monopoly power, and, with probability $1 - q$, the company will face no competition. Revenues from the drug sale will then remain at these new levels forever.2

Continuing to simplify, it is further assumed that the risk associated with the stream of revenues generated by the drug is fully diversifiable (i.e. it is unrelated to what happens in the broader economy). This allows for the discounting of the future cash flows at the risk-free rate of interest, which is assumed to equal 8 per cent.3 Given the above values, and noting that the expected revenues are equal to $400, it certainly seems that the company should go ahead with the project, since its

\[
NPV = -3200 + \sum_{t=0}^{\infty} \frac{400}{(1+0.08)^t}
\]

\[
= -3200 + 5400 = 2200.
\]

This conclusion, however, is premature if the company can defer investment. If the company waits for one year and invests only if the competition does not enter the market for the ulcer drug ($q = 0.5$), the NPV of the project is as follows:

\[
NPV = (0.5) \left[ \frac{-3200}{1.08} + \sum_{t=0}^{\infty} \frac{600}{(1+0.08)^t} \right]
\]

\[
= (0.5)[-2963 + 7500] = 2269.
\]
Clearly, it is better for the firm to wait a year before deciding to invest in the factory, since the project's NPV today is $2,269, whereas it is only $2,200 if it invests now.4

What is the value of having the flexibility to make the investment decision next year, rather than having to invest either now or never? The value of this timing option is just the difference between the two NPVs or $69. One can also say that the firm would be willing to pay $69 more for the opportunity to invest either now or next year over and above the opportunity that only allows it to invest now.

Let us briefly illustrate how the same option value can be arrived at by using the standard theory of financial options. Recall that we valued the investment opportunity in the ulcer drug at $2,269. Define $F_0$ to be the value today of the opportunity to invest in the ulcer drug, and $F_1$ the value of this investment opportunity next year. The value of $F_1$ depends on whether or not the competition is successful in developing and obtaining approval for their ulcer drug. Without competition, the annual revenues will be equal to $600, and the company will exercise its option by paying $3,200, receiving an asset which will be worth $8,100. The value of the investment next year assuming this is the state of the world is $F_1 = \sum_{t=0}^{\infty} 600/(1.08)^t - 3200 = $4,900. If the competition enters the market, however, annual revenues will fall to $200 and the option to invest will go unexercised. (This is so because exercising will result in a negative NPV since $\text{NPV} = \sum_{t=0}^{\infty} 200/(1.08)^t - 3200 = -$500.) In this case, $F_1$ will equal zero. Thus, we have all possible values for $F_1$ which enables us to find the value of $F_0$ by utilizing standard option hedging strategies.

The procedure is to construct a portfolio comprised of two components: the investment opportunity and a certain number of common shares (hereafter the twin security) that mimic the behaviour of revenues from the ulcer drug (i.e. their price is perfectly correlated with revenues from the drug). The twin security is currently trading at $100, with an equal probability ($q = 0.5$) to increase to $150$ or to decline to $50$. The number of twin securities to be included in the portfolio is to be selected so that the value of the portfolio next year is independent of whether revenues from the ulcer drug increase or decrease. (It will be seen below that this number will be negative implying we are short selling the twin security.) Since by construction the portfolio is risk-free, its return must equal the risk-free rate of 8 per cent. By setting the portfolio's return equal to the risk-free rate, one can easily calculate the current value of the investment opportunity.

Specifically, consider a portfolio in which one holds the investment opportunity, and sells short $n$ twin securities. The value of this hedged portfolio today is $H_0 = F_0 - P_0 n = F_0 - 100 n$, where $P_1$ is the price of the twin security at $t$. The value of the portfolio next year depends on the price of the twin security (and, by implication, the revenues from the ulcer drug) and is equal to $H_1 = F_1 - P_1 n$. If $P_1$ is equal to $150$, $F_1 = $4,900 and $H_1 = 4900 - 150 n$. If $P_1$ is $50$, however, $F_1 = 0$ and $H_1 = -50 n$. One can choose $n$ so that $H_1$ is independent of the revenues from the ulcer drug. To equalize terminal portfolio values we require:

$$4900 - 150 n = -50 n$$

or, $n = 49$. With $n$ set this way $H_1 = -$2,450 regardless of the revenues from the ulcer drug or the price of the twin security.

What is the return from holding such a portfolio? The return is the capital gain $H_1 - H_0$, minus any costs associated with holding the short position. Since the expected capital gain on the twin security is zero (the expected price next year is $100 which is also the current price), no rational investor would hold a long position unless she could expect to earn at least 8 per cent (i.e. the risk-free rate). Namely, selling the twin security short will require a payment of $0.08 P_0 = 0.08 \times 100 = $8 per share. Since our portfolio has a short position of 49 securities, the total payment to be made by the short seller equals $392. The return on holding this portfolio for a year is therefore

$$H_1 = H_0 - 392 = H_1 - (F_0 - n P_0) - 392$$

$$= -2450 - F_0 + 4900 - 392 = 2058 - F_0$$

Because this return is risk-free, it must equal the risk-free rate of 8 per cent times the initial value of the portfolio, $H_0 = F_0 - 100 n$, or

$$2058 - F_0 = 0.08 (F_0 - 4900).$$

Thus, the value of the investment opportunity today ($F_0$) is equal to $2,269, which is the same
value as that obtained in the previous section by calculating the NPV of the investment opportunity under the assumption that we follow the optimal strategy of waiting a year before deciding whether or not to invest.

As this example shows, real options analysis allows decision-makers to correct standard NPV (which is flawed): the end result can be termed option-based NPV, where:

\[
\text{Option-Based NPV} = \text{Traditional NPV (the NPV of expected cash flows)} + \text{Option premium.}
\]

The advantage of this approach arises from its potential to quantify the option premium or the flexibility component of value. This does not mean that traditional NPV should be ignored; rather, it is a necessary component of option-based NPV -- traditional NPV is a special case of option-based NPV. Specifically, the ability of management to adapt to changes in the economic environment introduces skewness in the probability distribution of NPV by increasing the probability assigned to favourable outcomes and reducing the probability assigned to unfavourable outcomes.

The option to change inputs in response to relative price changes

An important case where standard NPV analysis breaks down is when a firm must choose between several fixed input-fixed output operating technologies and one that allows for substitution between inputs and outputs. Such substitution may be called for as market conditions change. NPV analysis however ignores such opportunities to switch gears and thus undervalues a flexible technology by ignoring the inherent real options that it affords.

One of the problems faced by Ontario Hydro and other provincially-owned power utilities is that new technology has made it easier and cheaper for big users of electricity to build small power plants to generate their own electricity. To make things more concrete, consider a mini-mill steel producer that is considering expansion, and requires more energy for this purpose. It can choose among three mutually exclusive facilities: a coal-burning plant, a natural gas burning plant and a third flexible plant that allows the firm to switch back and forth between energy-producing inputs depending on the evolution of their relative prices over time.\(^5\)

The capital budgeting decision of the firm will amount to comparing the construction costs of the three facilities to the present values of the input costs associated with their use. It is likely that the flexible generator will be the most costly to construct. Nevertheless this decision, properly made, must recognize, in the case of the flexible system, the valuable option to switch back and forth between power sources as relative prices fluctuate, which is the focus of this section.

For simplicity we will restrict ourselves to a two-period model. Suppose at present the costs of the two inputs are identical per unit of energy, at (say) $1. Next period, the cost of coal will either increase by k per cent with probability .5, or decrease by k per cent with probability .5.\(^6\) The same will occur in the following period.\(^7\) This being the case, it may be in the interest of the firm to switch from one power source to another a number of times. In the absence of switching costs, the firm will always use the currently lowest-cost input.\(^8\)

The following tree diagram shows the evolution of energy costs over time (Chart 1).

It is straightforward to calculate the expected present values of the variable costs (VC) of the two fixed systems as:\(^9\)

\[
\text{VC}_{\text{gas}} = \text{VC}_{\text{coal}} = -1/1.1 - 1/1.1^2 = -$1.736 million
\]

When there are no switching costs, the expected value of a flexible system is the same as the value of a fixed coal generator plus the value of the option to switch to gas at any time, or, equivalently, the value of a fixed gas generator plus the value of the option to switch to coal at any time.

In particular, we have:

\[
E(F) = -(\text{VC}_{\text{gas}} - F(\text{gas} \rightarrow \text{coal})) = -(\text{VC}_{\text{coal}} - F(\text{coal} \rightarrow \text{gas})).
\]

where \(E(F)\) is the equity value of the flexible system net of expected revenues and fixed costs,\(^10\) and \(F(A \rightarrow B)\) is the value of the option to switch from A to B if advantageous.

Note that both of the options are multiple in the sense that the switching can occur in either of the two future periods, so we have:
Without switching costs the general problem reduces to a series of simpler myopic problems with overall value being calculated as the sum of the separate component values. The problem is that it is not credible that switching from one energy source to another can be done without any cost. When such costs exist, the separate switching options are no longer independent and the separate values no longer add up to the combined flexibility value. Let $S(A \rightarrow B)$ equal the switching cost in moving from energy source $A$ to $B$. If the firm enters a period using source $A$ but prices dictate a move to $B$, $S(A \rightarrow B)$ is a one-time cost that must be incurred.

Suppose, in moving from gas to coal, the cost is $50,000$, while in moving from coal to gas the cost is $10,000$. That is, we have $S(gas \rightarrow coal) = .05$ and $S(coal \rightarrow gas) = .01$. Now let us recalculate the tree diagram of cost savings in going from gas to coal, or from coal to gas (Chart 3).

Notice the difference between the above cost savings tree and that relevant when there are no switching costs. For each cell, we must subtract $0.05$ or $0.01$. If the value becomes negative, we set the cell entry to zero, since one is not forced to switch.

It is easy to see that the value of one fixed system plus the option to switch to the other energy source when advantageous is no longer identical to that based on beginning with the other input. That is, we have

$V_{coal} + F(coal \rightarrow gas) = V_{gas} + F(gas \rightarrow coal)$.

The reason is that the two fixed systems have identical values but now the two compound switching options have different values. The determination of cost is not so simple now because, in the presence of switching costs, the flexible system’s value can no longer be viewed as the sum of either of the fixed system’s values plus the compound option to switch to the other system at will.

With switching costs, the exercise of a previous option creates a series of nested new options analogous to a compound option. To solve the problem we must use a process of (backward) dynamic programming. In such cases the value of the flexible system must be determined simultaneously with the optimal energy source.

Let us consider the company’s problem. At any point in time, it must decide which energy

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**Chart 1**

Tree diagram for energy costs (gas, coal)

<table>
<thead>
<tr>
<th>t=0</th>
<th>t=1</th>
<th>t=2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1, 1.21</td>
</tr>
<tr>
<td></td>
<td>1, 1.1</td>
<td></td>
</tr>
<tr>
<td>1, 1</td>
<td></td>
<td>1, 0.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1, 0.81</td>
</tr>
</tbody>
</table>

**Chart 2**

Tree diagram of cost savings (from gas to coal; coal to gas)

<table>
<thead>
<tr>
<th>t=0</th>
<th>t=1</th>
<th>t=2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0.00; 0.21</td>
</tr>
<tr>
<td></td>
<td>0.00; 0.10</td>
<td></td>
</tr>
<tr>
<td>Gas; Coal</td>
<td></td>
<td>0.01; 0.00</td>
</tr>
<tr>
<td></td>
<td>0.10; 0.00</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.19; 0.00</td>
</tr>
</tbody>
</table>

\[ F(\text{coal} \rightarrow \text{gas}) = F_1(\text{coal} \rightarrow \text{gas}) + F_2(\text{coal} \rightarrow \text{gas}); \]

\[ F(\text{gas} \rightarrow \text{coal}) = F_1(\text{gas} \rightarrow \text{coal}) + F_2(\text{gas} \rightarrow \text{coal}). \]

To confirm the above result, consider the cost saving in moving from fuel source A to fuel source B at time $t$, if it is beneficial to do so. The tree diagram above shows the cost savings inherent in moving from gas to coal, or from coal to gas (Chart 2).

Focusing on the option to move from gas to coal, the value of the first period’s option is simply the discounted weighted average of the two option realizations, as follows:

\[ F_1(\text{gas} \rightarrow \text{coal}) = [.5^*0 + .5^*.1]/1.1 = 0.0455 \]

Similarly, for the period-2 option we have:

\[ F_2(\text{gas} \rightarrow \text{coal}) = [.25^*0 + .5^*.01 + .25^*.19]/1.1^2 = 0.0434 \]

The combined value of the option is then:

\[ F(\text{gas} \rightarrow \text{coal}) = 0.0455 + 0.0434 = 0.0889 \]

To interpret, ignoring differential construction costs, the flexible system increases the NPV of the project by $89,000 relative to either of the two fixed systems.\textsuperscript{11}
source to use. Should it continue with coal for example, or should it move to gas? A switch will be optimal only if the value from switching immediately exceeds the value from delaying potential switching. That is, the value of the flexible system at t given you are in state s operating with energy source A is:

\[ V_t^s(A) = \max \{ c_t^s(A) + E[V_{t+1}^s(A)]/(1 + r), \]
\[ c_t^s(B) + E[V_{t+1}^s(B)]/(1 + r) - S(A \rightarrow B) \]

where \( E[V_{t+1}^s(i)] = p^t E[V_{t+1}^s(i)] \)
\[ + (1-p^t) E[V_{t+1}^s(i)] \]

where \( c_t^s(i) \) is the cash flow at t using source i assuming state of the world s. Note that '+' represents the case where energy source i increases in price and '-' represents the case where its price falls. The expected future values are based on the assumption that the firm always acts in the optimal fashion in future periods. Because of this one must work backwards from the end. To arrive at the value of the flexible system, one can work with the following two decision trees (Chart 4).

The first tree gives for each node the present value of current and future energy variable costs assuming that the company has entered the node using gas; while the lower tree does the same assuming the company has entered using coal.

As just stated, we need to start at the end. The terminal (\( t = 2 \)) cell entries give the \( \max \{c_t^s(A), c_t^s(B) + S(A \rightarrow B)\} \). For example, looking at the upper tree, where it is assumed that we enter each period using gas, but may or may not switch in each node depending on what is optimal; in the '+' node, we would stay with gas since it is cheaper (1.21 vs. 1); in the '-' node, even though coal is cheaper (.99 vs. 1), we would not switch since the differential falls short of the switching cost (.05); and finally, in the '-' node, we switch to coal since the differential (.81 vs. 1) is sufficiently large relative to the cost of a switch. Notice that when we are at the last period the forward-looking part of the maximum expression does not come into play.

It will however matter for the other periods. To illustrate the procedure, consider cell (a) which is calculated as follows:

\[ \max\{-1 + [.5(-1) + .5(-1)]/1.10, -1.1 - 0.05 \]
\[ + [.5(-1.01) + .5(-0.99)]/1.10\] = -1.909

The goal is to calculate the value of the flexible operation assuming we have entered this node using gas as our energy source, and that all
The optimal investment scale with expansion options

Next we consider another option of interest to many companies, namely the option to expand operations. Often a firm is faced with the following problem. Forecasters are calling for an increase in demand necessitating an increase in capacity. It is usually the case that expansion can take place incrementally or in large leaps: for example, one large facility can be constructed, or several smaller facilities can be built over time as demand merits. What suggests the advisability of the former course of action may be due to economies of scale. One large facility is likely to cost less than three times that of three smaller facilities each producing one third the output of the large facility.

Real options theory may be able to clarify the debate. The advantage of starting small is that one retains valuable real options. If demand picks up the option to expand can be exercised, while if demand stays stagnant (or drops), the option is out of the money and will not be exercised. On the other hand, if one starts large, all options have already been exercised. Such an issue has special resonance for public utilities such as Ontario Hydro which, as mentioned earlier, undertook several large-scale expansion projects in recent decades, thereby surrendering its future flexibility.

Once again we provide a concrete example, which, though idealized, has the ability to confer some salient insights. Suppose a utility must respond to new demand. Currently this reflects the opportunity to increase cash flows at the margin by $1 billion. One choice is to build a small facility for $5 billion which would be able to satisfy demand up to this level but not above. Currently, it is unknown whether the additional demand will remain at the current level of $1 billion. If the economic expansion is to continue, it is predicted that demand will increase by 70 per cent. If the expansion slows down, demand is expected to drop by 20 per cent over the next year. In the following year (year 2), demand will further increase by 70 per cent or drop by 20 per cent. From this point on, it is believed that demand will be stable. For simplicity, we assume

future decisions are optimally made. We compare the cost associated with staying with gas which takes into account that we will enter the next period using gas as our energy source, with that associated with moving to coal. If we stay with gas the immediate cost is 1 and the present value of expected future costs assuming optimal future behavior (switching where appropriate) is -1/1.10. The sum is 1.909. This value is compared with that associated with switching to coal. If we switch to coal the cost of coal is 1.1; the switching cost is .01; and the present value of expected future costs assuming optimal future behavior is [.5(1.01) + .5(.99)]/1.10 (from the lower tree). The sum of the latter elements is obviously larger (i.e. a smaller absolute value) than the sum of the former. Thus the firm should stay with gas (assuming it entered using gas) and the present value of all costs as of this time and node is 1.909. In such fashion, each node's present value of current and future costs is calculated.

Looking at the values in the initial nodes (i.e. at t = 0) of the two tree diagrams, the implication is that the flexible system's present value of variable costs is 1.653. The dynamic programming solution also tells us that our flexible system should begin in the coal mode. Comparing this value for the flexible system to that of either of the fixed source systems, their present values of the variable costs are (in both cases):

VC = -1/1.1 -1/1.1^2 = -1.736

Since the flexible system has a lower cost, it is quite possible that the additional construction cost associated with the flexible system will be justified.

Interestingly, similar issues arise in the case when one considers cogeneration, which in recent years, due to increased concerns about pollution, has attracted renewed attention. In the case of cogeneration, output flexibility is the issue rather than input flexibility. Loosely speaking, cogeneration is defined to be the simultaneous production of both electricity and useful heat energy (in the form of steam or hot water) from a single system. Surprising as it is however, recent papers (e.g. Diener and Cain 1993) ignore the issues raised here in that they use traditional analysis when providing a financial perspective on cogeneration.
that any excess generating power cannot be sold outside of the utility’s own grid.\textsuperscript{15}

In anticipation of increasing demand, the utility can, for an extra $4 billion, build a medium-sized generating station with double the additional capacity (i.e., it can generate cash flows up to $2 billion). Alternatively, a large generating station that can triple the capacity (i.e., generate cash flows up to $3 billion) can be built at an incremental cost of $7 billion ($4 + 3$). These numbers reflect clear economies of scale. Suppose however that, if the utility decides to wait, doubling the capacity will require $5 billion (instead of the $4 billion if it builds it now), and tripling the capacity will cost $10 billion ($5 + 5$; instead of the $7 billion based on current construction). What is the best course of action today — to build a small, medium-sized or large facility?

To provide perspective, let us first calculate the present values of the three possible construction projects. These are to build small, medium-sized and large facilities. At first, we work in terms of expected cash flows without consideration of any expansion options. The following tree diagram will be helpful. These show the cash flows that can be generated by the projects in all possible states of the world (Chart 5).

Notice the differences. The initial investment varies with the size of the facility as described above. Regardless of the size of the facility, demand will evolve in the same stochastic fashion. The problem is that in the case of the small and medium-sized facilities capacity constraints will be binding at some point.

Assuming a risk-free rate of interest is 5 percent, the NPVs of the three projects are as follows:

\[
V_{\text{small}} = -5 + \{.5(1) + .5(.8))/1.05 \\
+ \{.25(1) + .5(1) + .25(.64))/[1 + 1/.05]/1.05^2 \\
= 13.21
\]

\[
V_{\text{medium}} = -9 + \{.5(1) + .5(.8))/1.05 \\
+ \{.25(1) + .5(1) + .25(.64))/[1 + 1/.05]/1.05^2 \\
= 17.71
\]

\[
V_{\text{large}} = -12 + \{.5(1) + .5(.8))/1.05 \\
+ \{.25(1) + .5(1) + .25(.64))/[1 + 1/.05]/1.05^2 \\
= 18.95
\]

Clearly, if one uses the traditional NPV, which ignores the expansion options, it is preferable to go ahead and immediately build a large facility.

<table>
<thead>
<tr>
<th>Chart 5</th>
<th>Tree diagram for cash flows -- small/medium/large generator</th>
</tr>
</thead>
<tbody>
<tr>
<td>(t=0)</td>
<td>(t=1)</td>
</tr>
<tr>
<td>(-5; -9; -12)</td>
<td>(1; 1.7; 1.7)</td>
</tr>
<tr>
<td>(0.8; 0.8; 0.8)</td>
<td>(0.64; 0.64; 0.64)</td>
</tr>
</tbody>
</table>

Of course the true value must take into consideration the expansion options inherent in the selection of either the small- or medium-sized facilities. The solution procedure for option-based NPV is similar to that described in the previous section, once we recognize the fact that different-sized facilities are tantamount to different operating modes. Since we have three different possible scales, in effect we have three different operating modes (unlike the case in the previous section where we had only two fuel sources to consider).

What about switching costs in moving from one mode to the next? Given our assumptions, these are as follows: \(S(\text{small} \rightarrow \text{medium}) = 4; S(\text{small} \rightarrow \text{large}) = 7; S(\text{medium} \rightarrow \text{large}) = 3; S(\text{large} \rightarrow \text{small}) = S(\text{medium} \rightarrow \text{small}) = \infty.\)

Note that we have made the implicit assumption that one cannot move from a large facility to a smaller facility.\textsuperscript{16}

Consider the utility's problem. At any point in time, the utility must decide whether to keep its current capacity or to expand. An increase in capacity will be optimal only if the value from switching immediately exceeds the value from delaying potential switching. The value of a facility at time \(t\) given you are in state \(s\) operating at scale \(A\) is:

\[
V^s_t(A) = \max \{ c_t^A(A) + E[V_{t+1} E^s(A)]/(1+r), c_t^B + E[V_{t+1} E^s(B)]/(1+r) - S(A \rightarrow B), c_t^C + E[V_{t+1} E^s(C)]/(1+r) - S(A \rightarrow C) \}
\]

where: \(E[V_{t+1} E^s(i)] = p^*E[V_{t+1} E^s(i)] + (1-p^*)E[V_{t+1} E^s(i)]\)
where \( c_t^i(i) \) is the cash flow at \( t \) using scale \( i \) assuming state of the world \( s \). Once again, note that ‘+’ represents the case where demand increases and ‘-’ represents the case where it falls. As before, since the expected future values are based on the assumption that the utility always acts in the optimal fashion in future periods, one must work backwards from the end. To arrive at the value of the system where expansion options exist, it is helpful to work with following three decision trees (Chart 6).

Note that the calculations are done in the same manner as described in more detail in the previous section. Also, it is apparent that the NPV of the flexible system, when one begins large, is the same as for a fixed system. This of course is due to the fact that once the utility constructs the large facility, there are no unexercised options remaining. Nevertheless in the other two cases ignoring options leads to a substantial downward bias in value estimation: in the case of the medium-sized generator the error is roughly 15 per cent while in the case of the small plant it is 38 per cent.

The overall conclusion is that, unlike the preferred alternative based on traditional NPV, the utility should begin with a small facility and make changes when demand warrants. In such a way it maximizes the value of the project. If utilities ignore expansion options and their impact on project value, there will clearly exist a bias in favor of overly large production facilities. One can only wonder if such traditional tools are at the root of Ontario Hydro’s current excess capacity problems.

**Perspective**

In our previous analysis, to make the discussion more tractable, a series of simplifying assumptions were made. For example, we only allowed uncertainty to remain unresolved for several periods. Nevertheless, an extension to multiperiod scenarios can be operationalized exactly along the lines mapped out here. Doing so does not alter the essentials of our analysis and conclusions. In addition, in order to focus on the “decision tree” and option-like nature of the problem, the analysis was simplified by assuming that the risk over the future cash flows was fully diversifiable (that is, unrelated to events in the overall economy). In reality, how-
ever, projects with embedded options typically call for discount rates that change endogenously with the value of some underlying state variable. The natural extension of the analysis is, therefore, to introduce a modification to the solution technique that accounts for asset valuation when risk is not fully diversifiable.

In general, one can continue to use the risk-free rate of interest for the purpose of discounting as long as one adjusts the actual drift rate of the driving stochastic variable (i.e. the cash flows of the project) by a certainty equivalent drift rate. This adjusted drift equals the actual drift on the variable minus the risk premium that would emerge in market equilibrium on an asset with the same risk features as the stochastic variable. Intuitively, instead of discounting the actual expected cash flows at the risk adjusted interest rate, one can equivalently discount at the risk-free interest rate and adjust the expected cash flows for risk. In the context of dynamic programming, instead of using the actual decision tree to discount expected optimal future cash flows (computed using the actual probabilities) at the required risk adjusted discount rate, real options valuation uses an equivalent risk-neutral decision tree, discounting expected optimal cash flows (computed using risk-neutral probabilities) at the risk free rate.

How does one derive these adjusted or risk neutral probabilities that allow the expected values to be discounted at the risk free rate? Under very specific conditions where one can identify a security with cash flows perfectly correlated with the cash flows of the project, the methodology employed in section 2 using a twin security, together with an equilibrium asset pricing model (e.g. the capital asset pricing model), can be used in order to derive the appropriate discount rate (Mason and Merton, 1985). On the other hand, in cases where one is unable to identify a traded security with the desired properties (as is often true when one deals with investments in real assets), the question of which discount rate should be used is more complex. The required techniques are beyond the scope of this paper and the interested reader is referred to Kulatilaka (1995a, 1995b).17

In the three previous sections we highlighted several potential real options. As companies in a broad range of industries are learning, opportunities to apply real options theory to investments are numerous. To conclude, a few further examples may be instructive.

Many natural resources are subject to large fluctuations in prices. A gold mining company, for example, may consider a temporary shut down of its mining operation in cases when revenues fall below the variable cost of production. The operator of the gold mine can be thought of as having a call option on the price of gold. When the price of gold increases, the operator profits. When gold price declines, losses are limited. In general, if market conditions are more favorable than expected, the firm can expand the scale of production or accelerate resource utilization. In cases where market conditions are less favorable than expected, management can reduce the scale of operation, or, in extreme cases, production may be halted and restarted only when prices recover.18

Another problem with the traditional NPV rule is that it ignores the value of creating options. Sometimes an investment that appears uneconomical when viewed in isolation may, in fact, create options that enable the company to undertake other investments in the future should market conditions turn favourable. An example is research and development. By not accounting properly for the options that R&D investments may yield, naive NPV analyses lead companies to invest too little. Alternatively, one often observes that some managers are intuitively aware of this problem and appear to over-ride the results from the NPV rule. For example, entrepreneurs sometimes invest in seemingly risky projects that would be difficult to justify by a conventional NPV calculation using an appropriate risk-adjusted cost of capital. Such projects generally involve R&D or some other type of exploratory investment. In other words, the exploratory investment creates a valuable option. Once the value of the option is reflected in the returns from the initial investment, it may turn out to have been justified, even though a traditional NPV calculation would not have found it attractive. In short, an early investment (e.g. R&D, a lease on undeveloped land or oil reserves, a strategic acquisition, etc.) is a prerequisite or a link in a chain of interrelated projects, opening up future growth opportunities. Such projects can often be found in industries where strategic actions are important for long term sur-
vival (e.g. high-tech industries, computers, pharmaceutical).¹⁹

Many modern production facilities provide management with the capability of shifting the output mix in response to price or demand changes (i.e. product flexibility). For example, many computer assembly lines can produce a diverse mix of different laptop computers in response to changing demand. Refineries often can produce a diverse mix of outputs (e.g. heating oil, gasoline, etc.) in response to changes in relative output prices. Product flexibility does not arise naturally. Management acquires this option for a price at the design phase of the project.²⁰

In the examples provided above, the focus was on individual real options and the insights that one can derive when real options theory of investment is applied. Real-life projects, however, are often more complex in that they may involve a number of real options whose value may interact. Trigeorgis (1993) identifies situations where option interactions can be small or large, negative or positive. He illustrates the importance of properly accounting for interactions among the options to defer, abandon, contract or expand investment, and switch use. The main conclusion of his analysis is that the combined value of a portfolio of real options may differ from the sum of separate option values.

Notes

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1. For a detailed discussion and abundant references on real options see Dixit and Pindyck (1994) and Trigeorgis (1996).

2. The option value in this example is the lost value from committing too early. As pointed out by an anonymous referee, in some cases offsetting this is the additional value associated with early deterrence when investment is made by the incumbent firm at an earlier stage than a prospective entrant. Nevertheless this can be incorporated into the above analysis by the appropriate alteration of cash flows and probabilities. For a theoretical discussion of this issue, see Alvazian and Berkowitz (1992).

3. This issue is discussed more fully in section 5.

4. If there is no option to wait a year, and hence no opportunity cost to killing such an option, the standard NPV rule applies. Similarly, if in the following year the firm can disinvest (i.e. recover the $3,200) should the competition enter, the firm should invest today.

5. This example is somewhat similar to that of Kularilaka (1993).

6. One might be tempted to suggest that the firm wait one year in order to have the uncertainty resolved. To abstract from this timing option, we assume that one of the three generators must be built immediately.

7. Expanding the example to a large number of periods is straightforward both conceptually and computationally. Deaves and Krinsky (1997), for example, solve a 50-period problem with varying costs using a personal computer.

8. As we will see, in the presence of switching costs, however, this need not be true if the switching costs outweigh the advantage of reduced energy prices.

9. Note that .5*1.1+.5*.9=1 and .25*1.21+.5*1.99+.25*.81=1.

10. Note that the revenues are the same for all systems, so it is safe to ignore them; as for the initial system costs, these would of course be compared once the system with the lowest variable cost is identified.

11. Note that the same result can be obtained by assuming that we started with a coal-burning system but can costlessly move whenever we like to gas.

12. That is, $\text{F}(A \rightarrow B) = \text{F}_1(A \rightarrow B) + \text{F}_2(A \rightarrow B)$.

13. In particular, in the case of moving from gas to coal we have:

\[ \text{F}_1(\text{gas} \rightarrow \text{coal}) = (0.50 + 0.50) / 1.1 = 0.2278 \]

\[ \text{F}_2(\text{gas} \rightarrow \text{coal}) = (0.25*0.20 + 0.50*0.25) / 1.1^2 = 0.0289 \]

\[ \text{F}(\text{gas} \rightarrow \text{coal}) = 0.0227 + 0.0289 = 0.0516 \]

Similarly, in moving from coal to gas we have:

\[ \text{F}_1(\text{coal} \rightarrow \text{gas}) = (0.50*0.09 + 0.50) / 1.1 = 0.0409 \]

\[ \text{F}_2(\text{coal} \rightarrow \text{gas}) = (0.25*0.20 + 0.50*0.25) / 1.1^2 = 0.0413 \]

\[ \text{F}(\text{coal} \rightarrow \text{gas}) = 0.0409 + 0.0413 = 0.0822 \]

14. The cell calculations for $t = 1$ and $t = 0$ are as follows:

(b) $\text{max} \{ 1 + 0.5 \times (-0.86) / 1.1 \} = 0.905 + 0.5 \times (-0.86) / 1.1 = -1.768$

(c) $\text{max} \{ 1 + 0.5 \times (-1.01) / 1.1 \} = 0.899 + 0.5 \times (-1.01) / 1.1 = -1.919$

(d) $\text{max} \{ 1 + 0.5 \times (-1.01) / 1.1 \} = 0.899 + 0.5 \times (-1.01) / 1.1 = -1.718$

(e) $\text{max} \{ 0.5 \times (-1.909) + 0.5 \times (-1.768) / 1.1 \} = -0.045 + 1.5 \times (-1.869) + 0.5 \times (-1.768) / 1.1 = -1.671$

(f) $\text{max} \{ 0.5 \times (-1.909) + 0.5 \times (-1.768) / 1.1 \} = -0.045 + 1.5 \times (-1.869) + 0.5 \times (-1.768) / 1.1 = -1.909$

15. It is true that electrical generating capacity does have a value in the secondary market. On the surface then the flexibility is symmetric. Nevertheless, it is clear that there will be additional costs incurred when such power is geographically transferred (e.g. the power loss that occurs is a positive function of distance and the efficiency of the grid). Thus, even taking such an
issue into account will still not lead to perfect symmetry. Altering our example to accommodate this issue would unduly complicate the computational complexity of the example.

16. Assuming that there are no additional fixed costs one of course would not want to consider scaling back, it is straightforward to extend the analysis so as to incorporate mothballing (and its attendant cost).

17. Kulatilaka takes into account the fact that the underlying stochastic variables are likely to be the prices of non-traded assets, in which case standard arbitrage arguments are inappropriate to value real options. Instead, relying on the Cox, Ingersoll and Ross (1985) extension of the risk neutrality argument, he adjusts for rate of return shortfalls in the prices of non-traded assets. It is important to note that in the latter case one has to resort to the use of an equilibrium model (e.g. CAPM) to obtain the shortfall in the growth rate. For a practical application, see also Kulatilaka (1993).

18. See, for example, Brennan and Schwartz (1995) and Trigeorgis and Mason (1987).


References


