Naive Versus Conditional Hedging Strategies: The Case of Canadian Stock Index Futures

Richard Deaves
McMaster University

Abstract
The hedging performance of Canadian stock index futures, namely the now defunct TSE 300 contract and the TSE 35 currently trading on the Toronto Futures Exchange, is investigated since inception in early 1984. Results utilizing a conditional hedging strategy are compared to more naive procedures. No value added is apparent for the more complex methodology. This is probably due to the Canadian market's high degree of arbitrage efficiency and the monthly contract cycle, both in effect after the first year of trading.

Résumé
Cette étude porte sur l'efficacité d'opérations de couverture reposant sur le marché à terme canadien sur indices boursiers depuis 1984. Mon analyse inclut le défunt contrat sur l'indice TSE 300 ainsi que son successeur, l'indice Toronto 35. Je compare des stratégies de contrepartie conditionnelles à des stratégies naïves et conclus que ces dernières affichent une performance tout aussi bonne. Il est fort possible que ce résultat soit dû au haut niveau d'efficacité du marché canadien ainsi qu'au cycle de maturité mensuel adopté par la bourse de Toronto un an après l'ouverture du marché.

Stock index futures have enjoyed great success since their inception in 1982. In large part this success has been due to the satisfaction of a demand on the part of hedgers for a low transaction cost vehicle for the purpose of adjusting market exposure. The proper determination of hedge ratios and the efficacy of hedging based on these ratios have been a central concern for practitioners as well researchers. For example, it is well known that contract "maturity" has an impact on optimal hedge ratios (e.g., Lee, Bubyns, & Lin, 1987; Merrick, 1988).

In addition, Merrick (1988) has shown that hedge efficacy is diminished by futures mispricing, where the latter is defined to be the difference between the observed futures price and that implied by strict adherence to the carry cost pricing model. Indeed the carry cost model only serves to provide a range of (arbitrage-efficient) prices (for the U.S., see Merrick, 1988; Modest & Sundaresan, 1983; Peters, 1985; for Japan, see Bailey, 1989; Brenner, Subrahmanyam, & Uno, 1989; for Canada, see Beyer, 1985; Deaves, 1990b). This is so, principally because the arbitrage mechanism required to enforce carry cost entails no significant costs and risk.[1]

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Address all correspondence to Richard Deaves, School of Business, McMaster University, Hamilton, ON, Canada, L8S 4M4.

The purpose of this paper is to investigate the hedging performance of Canadian stock index futures contracts, and to investigate to what extent performance is enhanced by employing subtle procedures over naive ones. Though hedging using U.S. index contracts has been extensively researched (e.g., Figlewski, 1984; Graham & Jennings, 1987; Junkus, 1987; Junkus & Lee, 1985; Lee, Bubyns, & Lin, 1987; Merrick, 1988), the hedging performance of Canadian index futures contracts traded on the Toronto Futures Exchange is as yet unexplored. The TSE 300 contract traded from January 1984 to June 1987, at which time it was phased out in favor of a contract on the more narrowly based TSE 35.[2] These contracts provide a new data set for testing the relative efficacy of various hedging methodologies. Of particular interest is a conditional hedging strategy, since such an approach is theoretically appropriate, as shown below. Since a conditional approach entails a movement towards complexity and additional analysis costs, it is useful to investigate the associated value added. Myers (1991) has shown, in the context of commodity futures, that a time-variant strategy leads to little improvement. Merrick (1988), using a somewhat different methodology from that employed here, also found little improvement for U.S. stock index futures.

The next section provides the appropriate theoretical background. The hedging methodologies and the empirical results are then described. Finally, the main findings of the paper are summarized.

Theoretical Background

The common practice of considering the minimum-variance hedge will be employed here. As shown by Fortin and Khoury (1988), such an approach is only strictly valid when a hedger has strong risk aversion. Thus, hedging should be properly viewed in a mean-variance context. Still, the minimum-variance hedge can serve as a good benchmark for the efficacy of partial (or over-) hedging strategies.

An investor wishing to hedge market risk in an arbitrary portfolio, whose market value at t is V_t, over a hedge horizon from t to $t+T$, sells futures contracts with specification for final cash settlement at $t+T+\tau$ where $\tau \geq 0$. Let m equal the futures contract multiplier; n_t equal the number of contracts sold at t; and I_t equal the level of the cash market index at t. Define the hedge ratio ($h_{t,\tau}$) as

$$h_{t,\tau} = \frac{n_t m I_t}{V_t} . \tag{1}$$

The hedge ratio is simply the ratio of futures "value" to cash market value.

Next, using (1), the hedged portfolio return (R_t^{ph}) can be written as

$$R_t^{ph} = R_t^p - h_{t,\tau} R_{t,\tau}^f , \tag{2}$$

where

$R_t^p = (V_{t+T} - V_t + D_{t,t+T}^p) / V_t$

$D_{t,t+T}^p$ = compounded dividends paid on the portfolio from t to $t+T$, valued as of $t+T$

$R_{t,\tau}^f$ = the futures "return" over the hedge period $= (F_{t+T,\tau} - F_{t,T+\tau}) / I_t$ [3]

$F_{t,j}$ = the index futures price at t for final cash settlement j periods ahead.

The minimum-variance hedge ratio is calculated by taking the variance of R_t^{ph} in (2), and minimizing with respect to $h_{t,\tau}$. The familiar result is

$$h_{t,\tau}^* = \frac{cov(R_{t,\tau}^f, R_t^p)}{var(R_{t,\tau}^f)} . \tag{3}$$

For simplicity the hedge ratio above is often viewed as maturity-invariant (not varying with τ) and time-invariant (not varying with t). This is indeed the traditional approach (as in Ederington, 1979). It is now well recognized that there is no reason why the covariance and variance need be constant. Myers (1991), for example, in the context of commodity futures, employed a GARCH methodology (see Bollerslev, 1986) to model conditional heteroscedasticity. Index futures call for a different approach based on a reformulation of (3).

In this regard, it is appropriate to decompose the returns in (3) into their constituent parts. First, assume that the market model is a reasonable return generating model for the portfolio return, and that the index is a reasonable proxy for the market. Therefore we can write

$$R_t^p = \alpha_p + \beta_p R_t^I + u_t^p, \tag{4}$$

where

R_t^I = the index return $= (I_{t+T} - I_t + D_{t,t+T}^I) / I_t$ and

$D_{t,t+T}^I$ = compounded dividends on index (in index units) from t to $t+T$, valued as of $t+T$.

Next, let us partition futures prices into those that would be observed if the theoretical carry cost model always held precisely ($F_{t,j}^*$), and residual mispricing ($M_{t,j}$), namely

$$F_{t,j} = F_{t,j}^* + M_{t,j} . \tag{5}$$

The theoretical carry cost price is defined to be

$$F_{t,j}^* = I_t (1+R_t)^j - D_{t,t+j}^I , \tag{6}$$

where

R_t = the risk-free rate as of t (assumed constant up to $t+T$).

It should be noted that the theoretical carry cost price can only be estimated, because future dividends and interest rates are unknown ex ante. The closer one is to contract maturity, however, the more accurate such estimates will be, since interest rates are not likely to change dramatically over a few days, and dividends up to a week or so in the future have generally already been announced.

Finally, actual futures returns are the sum of theoretical futures returns and "mispricing returns" ($R_{t,\tau}^m$), or

$$R_{t,\tau}^f = R_{t,\tau}^{f*} + R_{t,\tau}^m , \tag{7}$$

where

$R_{t,\tau}^m = (M_{t+T,\tau} - M_{t,T+\tau}) / I_t$ and

$R_{t,\tau}^{f*} = (F_{t+T,\tau}^* - F_{t,T+\tau}^*) / I_t$.

On substituting (6) into the definition of $R_{t,\tau}^{f*}$, straightforward manipulation allows the following convenient reformulation[4]:

$$R_{t,\tau}^{f*} = (1+R_t)^\tau [R_t^I - (1+R_t)^T - 1] \tag{8}$$

This means that, in the absence of mispricing, the futures return is equal to the ex post market risk premium multiplied by the gross τ-period risk-free rate.

Finally, substituting the returns in (4), (7), and (8) into (3), and assuming that all relevant covariances are zero[5], allows the following reformulation[6]:

$$h_{t,\tau}^* = \frac{(1+R_t)^\tau \beta_p \mathrm{var}(R_t^I)}{var(R_{t,\tau}^m) + (1+R_t)^{2\tau} var(R_t^I)} . \tag{9}$$

If $\tau=0$, unpredictable mispricing is precluded ($var(R^m_{t,\tau}) = 0$), and $h^*_{t,\tau} = \beta_p$. Usually, hedging must rely on contracts that will have time remaining as of hedge wind-up. If $\tau>0$, but there is no mispricing (or if the mispricing return is entirely predictable), $h^*_{t,\tau} = (1+R_t)^{-\tau}\beta_p$. The greater the futures contract maturity (τ), the more volatile futures are relative to cash prices, and the less hedging one need undertake. Even for a given τ, the hedge ratio can be time-variant since the risk-free rate will vary over time.

In addition, mispricing variability suggests a downward adjustment in the hedge ratio. The greater is such variability, the less effective will be the hedge, so less hedging will be done. It is variability that is paramount. Although it is true that any predictable change in mispricing will have an impact on the mean of hedged returns, from the standpoint of minimum-variance hedging, however, it is only mispricing "surprises" that matter.[7] To be precise, one must focus on the variance of the component of $R^m_{t,\tau}$, which is orthogonal to obvious explanatory variables which are known ex ante, prime examples being initial mispricing and contract maturity. The first is related to the tendency, documented in Merrick (1988), for mispricing to be eliminated over time. As for contract maturity, due to the necessity for cash-futures convergence at the final cash settlement date of the futures contract, the lower the τ, the greater one might expect this reversal tendency to be. In other words, mispricing variance is logically maturity-variant. Indeed, MacKinlay and Ramaswamy (1988) find that mispricing variability increases with τ.[8]

Hedge ratios may also be time-variant because the distribution of mispricing may vary over time. There is evidence that mispricing tends to diminish after a contract is introduced to the market (e.g., Peters, 1985), since a seasoning period may be required before market participants become accustomed to pricing relationships.

The common practice is to simplify by viewing the hedge ratio as constant. Then the hedge ratio and an ex post measure of hedging effectiveness can be simultaneously estimated. Inspection of (3) makes clear that the former is the slope coefficient in a simple regression of cash market returns on futures returns, while a measure of hedging performance is the R^2 from this regression.[9] The same influences that determine the conditional hedge ratio are present when (3) is estimated via regression, but the regression method averages out over the sample these factors. Whether much is lost by using this simplistic approach is an empirical question that will be addressed next.

Methodology and Results

Hedging Strategies

All Canadian index futures data utilized were Wednesday settlement prices for nearby contracts.[10] This allowed for the investigation of one week (Wednesday to Wednesday) hedges. The TSE 300 contract is the hedging vehicle from January 18, 1984, its first Wednesday of trading to June 17, 1987, its last, while the TSE 35 contract is used from June 17, 1987, to the last Wednesday in the sample, April 19, 1989.[11] The overall sample is partitioned into three subperiods: January/84-March/85, March/85-June/87, and June/87-April/89. The first break coincides with the change in the TSE 300 futures contract specification from a quarterly cycle (implying that nearby contracts have up to three months to go) to a monthly cycle (implying that nearby contracts have at most one month to run). The second break occurs after the phasing out of the TSE 300 contract in favor of the TSE 35 contract (which has always used a monthly cycle).

Hedging performance is explored by investigating reduction in return variability. The portfolios that are hedged are for simplicity the index portfolios themselves, the TSE 300 during January/84-June/87 and the TSE 35 during June/87-April/89. Full sample results are presented by using a composite index/hedge: The TSE 300/35 index portfolio is hedged against the TSE 300/35 futures contract up to/subsequent to June 1987.

Three hedge ratios are used. First, as a benchmark, is the naive approach, which sets the hedge ratio equal to β_p. In this case, since the portfolio to be hedged is the index itself, the hedge ratio is unity. Second, the regression method is used. Here again the assumption is of a constant hedge ratio. Using this method, hedge ratios are determined as the slope coefficients in regressions of cash market returns on futures returns. The hedge ratios so calculated range from .972 to .990. The lowest R^2 is .96. The tight fit puts us on notice that probably little else can be "explained" by more subtle procedures.

Conditional Hedge Ratio

Finally, a maturity-variant and time-variant conditional approach along the lines of (9) is employed. To operationalize, beginning-of-hedge risk-free interest rates,[12] the number of days until contract expiration as of the hedge wind-up date, and sample-specific index return estimated variances are substituted into (9).[13] The latter values are 1.819%, 1.608%, and 2.755% for the three subperiods respectively, and 2.117% overall. Not surprisingly, market volatility just preceding and subsequent to the "major correction" of October 1987 is substantially higher than during the first two subperiods.

Some care needs to be taken with the estimation of mispricing return variance. As noted earlier, the expected value of the mispricing return is likely to vary considerably from week to week. If the future is initially overpriced (underpriced), it is reasonable to anticipate a negative (positive) mispricing return due to the tendency for theoretical and actual futures prices to converge over time, and additionally, this tendency is likely to be accentuated the closer the hedge horizon is to the final settlement date. Assuming linearity, these likely determinants of mispricing return were used to explain mispricing returns via the following regression:[14]

$$R^{m}_{t,\tau} = \beta_0 + \beta_1 M_{t,T+\tau} + \beta_2(\tau_t * M_{t,T+\tau}) + \epsilon_t \,. \qquad (10)$$

The estimation results are provided in Panel (A) of Table 1. The full sample evidence strongly supports the explanatory power of these two factors. For example, in the case of the TSE 35 contract during June/87-April/89, a 1% initial overpricing typically led to a -0.61% mispricing return over the following week. Thus, much of the initial mispricing was erased over the week. As for the impact of contract maturity on mispricing reversal, the evidence here is weak in two of the three subperiods (in one of the two, even having the wrong sign), but

Table 1
Predicting Mispricing Variability

(A) $R^{m}_{t,\tau} = \beta_0 + \beta_1 M_{t,T+\tau} + \beta_2(\tau_t * M_{t,T+\tau}) + \epsilon_t$

Sample period	b_0	b_1	b_2	SEE	R^2	DW
Jan/84 – April/89	-0.00019 (1.08680)	-0.77407 (10.70000)	0.00785 (6.64920)	0.003	.343	2.21
Jan/84 – March/85	-0.00042 (0.79416)	-0.38155 (2.13920)	0.00231 (0.97796)	0.003	.201	2.15
March/85 – June/87	0.00003 (0.11420)	-1.09530 (7.30300)	0.02615 (3.08830)	0.002	.433	2.16
June/87 – April/89	-0.00006 (0.25308)	-0.60688 (2.8820)	-0.01500 (1.22720)	0.002	.442	1.95

(B) $\text{RESSQ}_t = \gamma_0 + \gamma_1\tau_t + \epsilon_t$

Sample period	g_0	g_1	SEE	R^2	DW
Jan/84 – April/89	0.03491 (3.53290)	0.00176 (4.84420)	0.111	.079	1.46
Jan/84 – March/85	-0.00387 (0.09740)	0.00212 (2.62790)	0.162	.106	1.18
March/85 – June/87	0.04627 (3.16610)	0.00090 (1.01550)	0.086	.009	1.69
June/87 – April/89	0.04417 (2.97430)	0.00113 (1.25690)	0.078	.017	1.97

Note. b_i are estimates of β_i and g_i are estimates of γ_i. RESSQ_t = squared residuals from the Panel (A) regressions times 10,000. All regressions are estimated with ordinary least squares. Absolute t statistics are presented below the coefficient estimates.

Table 2
Hedging Efficacy Results

Sample period	Unhedged index portfolio Mean ret.	Unhedged index portfolio SD	Hedged portfolios H1 Mean ret.	H1 SD	H2 Mean ret.	H2 SD	H3 Mean ret.	H3 SD
				(A) All observations				
Jan/84-Apr/89	21.705	211.729	12.794	32.534	13.025	32.052	12.789	32.106
Jan/84-Mar/85	9.667	181.871	11.037	32.246	11.023	32.194	10.695	32.482
Mar/85-Jun/87	39.917	160.776	10.551	32.079	11.305	31.767	11.173	31.716
Jun/87-Apr/89	9.301	275.452	16.648	33.245	16.440	32.276	16.456	32.535
				(B) Positive initial mispricing				
Jan/84-Apr/89	5.291	230.112	31.009	32.994	30.342	32.388	30.154	32.483
Jan/84-Mar/85	-37.520	165.842	30.848	32.989	30.155	32.400	28.745	31.111
Mar/85-Jun/87	29.071	154.598	25.176	37.078	25.283	36.587	25.101	36.532
Jun/87-Apr/89	-5.465	292.328	36.187	28.850	35.007	28.498	35.389	28.558
				(C) Negative initial mispricing				
Jan/84-Apr/89	29.601	202.472	4.031	28.497	4.694	28.421	4.435	28.438
Jan/84-Mar/85	19.105	185.015	7.075	30.919	7.196	31.073	7.085	31.825
Mar/85-Jun/87	41.597	164.152	3.871	27.244	4.919	27.237	4.811	27.200
Jun/87-Apr/89	20.785	263.791	1.452	28.274	1.999	27.447	1.730	27.594

Note. H1: naive one-to-one hedge; H2: regression-based hedge; H3: conditional hedge. Mean ret. denotes mean returns (in basis points), SD = standard deviation of returns (in basis points). Numbers of observations, from top to bottom, are 274, 60, 118, 96; 89, 10, 37, 42; 185, 50, 81, 54.

the full-sample estimated coefficient is significantly positive. On average, an additional day to maturity implies a reduction in the reversal of mispricing of 0.79 basis points.

Maturity-variation in $var(R^m_{t,\tau})$ is next investigated. To test for such heteroscedasticity, residuals from the above regressions were squared and in turn regressed on τ. Panel (B) of Table 1 shows the results. The full-sample estimation produced a highly significant t statistic (4.84), but only an implied increase of 1.26% in standard deviation for every additional month of contract maturity. Fitted values from these regressions (in Panel (B)) were used as estimates of conditional mispricing return variances and substituted into (9).[15]

Hedging Performance

Table 2 provides evidence on hedging performance. Focusing on the top row of Panel (A), which gives full sample mean returns and return standard deviations for the unhedged index portfolios, and the same index portfolios hedged with short index futures using the three approaches described above, it is clear that there is a little value added in moving from the naive one-to-one approach to the regression approach, but none at all in moving to the conditional methodology. Why is there only a little improvement in moving to the regression method? Since the final settlement date is never more than one month away from the end-of-hedge date, the appropriate reduction in the hedge ratio below unity from this effect is quite small. As for the lack of improvement obtained using the conditional methodology, which accounts for mispricing, the low degree of mispricing observed in these contracts suggests minimal improvement by carefully modelling this factor (see Deaves, 1990b, for evidence). Still, one must grant, it is puzzling to find no improvement.

In Panels (B) and (C) of Table 2, the observations are partitioned by the sign of the initial mispricing. Notice that when the initial mispricing is positive (implying that hedging involves selling overpriced futures), the mean hedged return in two of the three subperiods, as well as in the overall sample, exceeds the mean unhedged return. The reason, of course, is that if the futures contract is overpriced, the futures contract return is likely to be negative, which is beneficial if you are selling index futures. Thus, in this case, hedging improves return while drastically reducing risk. The opposite scenario exists, however, when the future is initially underpriced, as revealed in the lower panel.

This discussion suggests that practitioners would be wise to overhedge in the former case, when hedging costs are low, and underhedge in the latter case, when hedging costs are high. One could begin with an expression such as (10) and project mispricing returns over the life of the hedge. Then, using these, hedged portfolio return means and standard deviations could be calculated for different hedge ratios. The optimal hedge could then be selected on the basis of standard mean-variance analysis and one's risk tolerance.

Concluding Remarks

Practitioners will be interested to learn that there was no value added in moving from the standard regression hedge to a maturity-variant and time-variant approach. Although theoretically the latter, more subtle approach is valid, and one suspects its outright inferiority may be sample-specific, the monthly contract cycle and low degree of mispricing in Canada were cited as reasons why the results should not have been too surprising.

It should be stressed, however, that all these empirical results have been based on the use of settlement prices. To the extent that these are mid-market prices (i.e., prices mid-way between the bid and ask) and that TFE spreads are wide, hedging will be more costly, and less hedging will be undertaken, than indicated in this paper. This could account for the low volume in TSE 300 and TSE 35 index futures on the Toronto Futures Exchange. Further investigation is required for this issue. Also, as noted in Deaves (1990a), given the nature of mandated settlement prices in thin markets, researchers must be wary in interpreting results.

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Notes

1 Carry cost can break down because of risk. For example, except over very short horizons, dividends are not known with certainty; and the surrogate portfolio may not move in perfect synchronization with the index. Also, transaction costs and restrictions on the use of short sale proceeds are ignored by carry cost. Market inefficiency, of course, is another possibility.

2 The TSE 300 contract reached its trading peak in 1986 with sales of nearly 60,000 contracts, representing about 1.75 bil-

lion Canadian dollars. The TSE 35, introduced in April 1987, experienced vigorous trading throughout 1987, but, consistent with U.S. experience, trading declined somewhat after the market drop of October 1987, and has not yet recovered to previous levels.

3 $R^f_{t,\tau}$, though termed the index futures return, is more accurately a futures price change scaled by the index. The reason is that the cash outlay is indefinite and may be zero if the investor already possesses sufficient cash equivalents for margining purposes. Here cash flow and marking to market considerations are abstracted from, which is reasonable given the short-term nature of the hedges examined.

4 Begin with the definition of $R^{f*}_{t,\tau}$ and manipulate:

$$R^{f*}_{t,\tau} = (F^*_{t+T,\tau} - F^*_{t,T+\tau}) / I_t$$
$$= [I_{t+T}(1+R_t)^\tau - D^I_{t+T,t+T+\tau} - I_t(1+R_t)^{T+\tau} - D^I_{t,t+T+\tau}] / I_t$$
$$= (1+R_t)^\tau [I_{t+T} - I_t(1+R_t)^T + D^I_{t,t+T}] / I_t$$
$$= (1+R_t)^\tau [R^I_t - ((1+R_t)^{\mathrm{T}} - 1)].$$

5 It is logical that the relevant covariances would equal zero. First, a zero covariance between portfolio-specific return and market return is an implication of the market model; second, there is no a priori reason to believe portfolio-specific return should be correlated with mispricing return; and third, the in-sample correlation of mispricing returns and index returns is close to zero.

Substitution into (3) gives for $h^*_{t,\tau}$

$$cov[(1+R_t)^\tau[R^I_t-(1+R_t)^T-1]+R^m_{t,\tau}, \alpha_p+\beta_p R^I_t+u^p_t]$$
$$/ var[(1+R_t)^\tau [R^I_t-(1+R_t)^T-1]+R^m_{t,\tau}].$$

Using that $cov(R^I_t, R^m_{t,\tau}) = cov(R^I_t, u^p_t) = cov(R^m_{t,\tau}, u^p_t) = 0$ allows us to work out (9) easily.

6 Non-market risk does not affect $h^*_{t,\tau}$, despite the fact that it will still have an impact on $var(R^{ph}_t)$. In fact it will be the only source of hedged return variability when there is no mispricing. The derivation of the hedge ratio using mispriced futures is similar in some respects to that of Merrick (1988). Salient differences are that he ignores portfolio-specific return, and does not decompose futures returns into one part attributable to index returns and another part attributable to changes in mispricing.

7 The cost of hedging is affected, as discussed below.

8 MacKinlay and Ramaswamy explain this finding by noting that, though round-trip transactions costs remain unchanged, various risks, such as dividend uncertainty and tracking risk, play a larger role the greater the contract maturity.

9 It should be noted that the latter is a within-sample measure, and is thus not necessarily reflective of the reduction in portfolio return variability really available to hedgers. The same criticism can also be levelled at the conditional approach if (as here) it uses within-sample parameter estimates.

10 Occasionally holidays necessitated the use of adjacent day data.

11 Nearby Canadian stock index futures settlement prices were taken from the Toronto Futures Exchange's *Futures Daily Record Report* and the Globe and Mail's *Report on Business*. As for cash indexes, the TSE 300 Composite and Total Return Indexes were obtained from the TSE Monthly Review, while the comparable indexes for the TSE 35 were obtained on diskette from the Toronto Stock Exchange.

12 Canadian short-term risk-free interest rates were proxied by 30-day Government of Canada T-bill yields (obtained on diskette from the Bank of Canada).

13 After Engle (1982), it has become commonplace to model asset return variances as conditional on residuals from the recent past. To investigate the appropriateness of such ARCH (autoregressive conditional heteroscedasticity) modelling techniques in this context, LM tests were conducted. It can be shown that when residuals from mean returns are regressed on lagged residuals, R^2 times the sample size (TR^2) is distributed, under the null, as $\chi^2(k)$ where k is the number of lags. In each of the three subperiods, index return homoscedasticity could not be rejected.

14 Another way to view this regression is to begin with

$$R^m_{t,\tau} = \delta_0 + \delta_{1t} M_{t,T+\tau} + u_t.$$

The magnitude of δ_{1t} is hypothesized to be negatively related to τ. Again assuming linearity, one can write

$$\delta_{1t} = \theta_0 + \theta_1 \tau_t + \nu_t.$$

Substitution yields

$$R^m_{t,\tau} = \delta_0 + \theta_0 M_{t,T+\tau}$$
$$+\theta_1(\tau_t * M_{t,T+\tau}) + u_t + (\nu_t * M_{t,T+\tau}).$$

This is tantamount to equation (10) in the text (ignoring heteroscedasticity issues).

15 These fitted values are just predicted squared deviations from conditional mean mispricing returns, or estimated variances. Other specifications which seemed reasonable (for example, including either absolute or squared mispricing as an additional explanatory variable, or just utilizing an unconditional variance) in several cases performed slightly better in hedging, but the specification alluded to in the text was retained since it seemed slightly more *a priori* defensible.