A NOTE ON SPECULATIVE VERSUS ARBITRAGE OPPORTUNITIES FROM INDEX FUTURES MISPRICING: SOME CANADIAN EVIDENCE

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ABSTRACT

A number of stock index futures markets have offered profit opportunities for arbitrageurs in the early years of trading, but after a period of seasoning, such opportunities have become infrequent. This is not the same as saying that speculative opportunities were unavailable. This contention is argued, and illustrated for the case of Canadian stock index futures.

I. INTRODUCTION

One of the most successful financial innovations introduced in the 1980s has undoubtedly been stock index futures. Research has followed market popularity, with the pricing of contracts being extensively investigated by researchers. Of primary interest has been how closely actual prices have conformed to those predicted by the carry cost model. Whenever the divergence is too large to be explained by transaction costs and various arbitrage risks, it implies both market inefficiency and profit opportunities for arbitrageurs. The principal finding has been that, after a period of contract seasoning, few such opportunities remained

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available. This has been documented extensively for U.S. contracts (see Bhatt & Cakici (1990); Cornell (1985); Merrick (1988); Modest & Sundaresan (1983); and Peters (1985)). Of late Bailey (1989) and Brenner, Subrahmanyam, and Uno (1989, 1990) have provided similar such evidence for Japan; as have Beyer (1985), Chamberlain, Cheung, and Kwan (1989) and Deaves (1990) for Canadian contracts.

Nevertheless such *arbitrage efficiency* is only a necessary condition for market efficiency. It is important to consider whether, even in the absence of arbitrage opportunities, excess risk-adjusted returns could have been achieved. Such departures from *speculative efficiency* are conceivable, because speculative trading can be undertaken with substantially lower total transaction costs than arbitrage. This possibility is empirically investigated using Canadian index futures. Index futures began trading on the *Toronto Futures Exchange* in 1984 and volume, though low by U.S. standards, has been steady.[1] Section 2 provides the appropriate theoretical background. The next section describes the methodology and details the results. Section 4 concludes.

II. THEORETICAL BACKGROUND

Arbitrage arguments can be used to demonstrate the carry cost pricing relationship that, in a frictionless risk-neutral world, continuously determines the theoretical price of an index futures contract. This price at t ($F^*_{t,j}$), with final cash settlement j periods in the future, is a function of the current spot index (I_t), current and expected future single-period gross risk-free rates (R_t), and expected future dividends in index units on the index (D^I_t), as follows:[2]

$$F^*_{t,j} = I_t \prod_{i=0}^{j-1} E_t(R_{t+i}) - \sum_{i=1}^{j} E_t(D^I_{t+i}) \prod_{k=0}^{j-i} E_t(R_{t+k}) \,. \tag{1}$$

Letting $M_{t,j}$ be the deviation of the actual futures price ($F_{t,j}$) from its theoretical level ($F^*_{t,j}$), we have

$$F_{t,j} = F^*_{t,j} + M_{t,j} \,. \tag{2}$$

It is commonplace to view the upper bound on $|M_{t,j}|$ that is consistent with market efficiency as equivalent to the magnitude of the total transaction costs and risks faced by the lowest-cost arbitrageurs. The relevant transaction costs are round-trip futures and cash market commissions and market impact costs.[3]

A T-period long "futures return" beginning at t on contracts with final cash settlement at $t + T + \tau$ ($R^f_{t,\tau} = (F_{t+T,\tau} - F_{t,T+\tau})/I_t$ where $\tau \geq 0$) can be straightforwardly decomposed into that which holds if futures prices are always precisely determined by carry cost ($R^{f*}_{t,\tau} = (F_{t+T,\tau} - F^*_{t,T+\tau})/I_t$), and a scaled change in mispricing, or "mispricing return" ($R^m_{t,\tau} = (M_{t+T,\tau} - M_{t,T+\tau})/I_t$). One can show that $R^{f*}_{t,\tau}$ is equal to the ex post (T-period) market risk premium compounded to the end of the futures contract, or[4]

$$R^{f*}_{t,\tau} = \prod_{i=0}^{\tau-1} E_t R_{t+T+i}[R^f_t - \prod_{i=o}^{T-1} E_t R_{t+i} - 1] \,. \tag{3}$$

A long futures return ($R^f_{t,\tau}$) will be high if $R^{f*}_{t,\tau}$ is high, which will occur if the market performs well and/or (less obviously) if there are downward dividend revisions and upward interest rate revisions;[5] and if $R^m_{t,\tau}$ is high, which means that initially negative mispricing has dissipated, or that initially positive mispricing has increased.

Consider the beta of a long futures position. If mispricing returns are uncorrelated with index returns, it is clear that the beta of a (long) futures position is simply the index beta (i.e., unity) multiplied by the (expected) gross risk-free rate from T to $T + \tau$.[6] This implies that the excess return to a long futures position is merely the mispricing return, while the excess return for a short position is the negative of the mispricing return.

Whereas substantial mispricing tends to be partially reversed by arbitrage activity (Merrick, 1988), theory is virtually silent as to appropriate price movements within the bounds based on arbitrage transaction costs (MacKinlay & Ramaswamy, 1988).[7] Nevertheless, since one knows that by final settlement mispricing will be eliminated, over shorter intervals it is logical to expect partial mispricing reversal in anticipation.[8]

III. RESULTS USING CANADIAN INDEX FUTURES

To investigate the efficiency of index futures prices, I use Wednesday settlement prices of *Toronto Futures Exchange TSE* (Toronto Stock Exchange) 300 index futures from March 1985 to June 1987; and Toronto 35 index futures from June 1987 to April 1989 are used. June 1987 coincides with the phasing out of the '300' contract and the introduction of the more narrowly based '35.' Though admittedly transaction data are preferable for a better matching of cash market and futures market prices, such data do not exist for the Canadian market. Non-overlapping weekly returns are calculated.[9]

I will now investigate arbitrage efficiency and then proceed to speculative efficiency. Before undertaking these tasks, however, one must have some notion of what constitutes appropriate transaction costs. Merrick (1988), for the *S&P 500* contract, estimates total transaction costs of 0.65 percent, split 0.05%/0.60% between futures and the cash market. Beyer's (1985) estimate of 0.67 percent for Canada during the first year of trading is based on 0.42 percent for round-trip stock commissions and 0.25 percent for futures commissions. Based on conversations with those close to the Canadian market, 0.10–0.15 percent now seems reasonable for the futures spread and round-trip commissions.

The existence of mispricing beyond transaction costs incurred by arbitrageurs is evidence of arbitrage inefficiency. To calculate mispricing, theoretical futures prices are calculated using equation (1).[10] For the four years of the overall sample, the most extreme case of absolute percentage mispricing is only 0.76 percent. If one uses ±0.6 percent as a reasonable no-arbitrage mispricing interval, there are arbitrage opportunities on only seven occasions (in the present sample) during March/85–June/87, and none at all thereafter.[11] For March/85–

Table 1. Excess Returns for Speculators Employing Various Filters Based on Minimum Absolute Mispricing

March/85–June/87

Lower Bound	*Minimum Absolute Mispricing* None	0.1	0.2	0.3	0.4	0.5
0.6	0.096	0.148	0.213	0.295	0.362	0.350
	111	76	51	32	14	6
	0.044	0.087	0.141	0.209	0.211	0.071
None	0.126	0.187	0.260	0.350	0.442	0.486
	118	83	58	39	21	13
	0.070	0.120	0.182	0.258	0.301	0.285

June/87–April/89

Lower Bound	*Minimum Absolute Mispricing* None	0.1	0.2	0.3	0.4	0.5
None	0.171	0.213	0.284	0.340	0.443	0.357
	96	76	45	25	12	2
	0.114	0.148	0.197	0.222	0.283	−0.425

March/85–April/89

Lower Bound	*Minimum Absolute Mispricing* None	0.1	0.2	0.3	0.4	0.5
0.6	0.131	0.181	0.246	0.315	0.399	0.352
	207	152	96	57	26	8
	0.092	0.136	1.90	0.245	0.290	0.100
None	0.146	0.199	0.271	0.346	0.442	0.468
	214	159	103	64	33	15
	0.106	0.153	0.213	0.274	0.337	0.278

Notes: Each cell contains (from top to bottom) the mean excess return, the number of transactions triggered by a minimum absolute percentage mispricing filter, and the mean excess return minus two standard deviations; All returns are over one week and are in percentage form; Absolute mispricing is also in percentage form.

June/87 mean mispricing is −0.05 percent with a standard deviation of 0.27 percent (implying mean mispricing is significantly negative). For June/87–April/89 the mean is −0.01 percent with a standard deviation of 0.25 percent. It is fair to conclude that the Canadian market is moving towards arbitrage efficiency, as is the U.S. experience. This more recent Canadian mispricing evidence compares favorably to that witnessed in the U.S., as reported in Merrick (1988), namely a mean of 0.01 percent with a standard deviation 0.41 percent for the *S&P 500* contract for 1985–86.

Next, filter strategies are used to investigate the scope for earning excess (one-week) returns. These are tantamount to departures from speculative efficiency. Table 1 is based on the obvious strategy of going long in index futures when mispricing is negative, and short when it is positive, in anticipation of future mispricing reversals. A filter is used to avoid overly small absolute mispricing. The search for the "right" filter is based on the obvious tradeoff: the greater/smaller is the magnitude of initial absolute mispricing one uses as a filter, the more/less profitable such a strategy is likely to be on average; on the other hand, there will be fewer/more opportunities. Therefore it seems best to examine several filters.

For each subperiod, the mean excess return (i.e., mispricing return), the number of transactions, and the mean excess return less two standard deviations (from top to bottom in each cell) are provided for strategies employing various filter rules. The cutoff levels of absolute mispricing vary by column. The two rows (where applicable) correspond to the case where we limit ourselves to arbitrage-efficient observations (i.e., those such that absolute mispricing does not exceed 0.6), and to the case without any exclusions.

Referring to the table, to take an example, if a trader has restricted himself to a lower bound of 0.4 percent on initial mispricing during March/85–April/89, his mean return to pursuing such a strategy, based on all observations, is 0.442 percent on a weekly basis. Even after deducting 0.1–0.2 percent for futures transaction costs and subtracting two standard deviations from this value, he is left with a mean return of 0.137–0.237 percent or 7.4–13.1 percent on an annual basis. The strength of these results is only slightly reduced by omitting from consideration those mispricing opportunities which were probably large enough to have elicited arbitrage activity. If the trader applies the 0.4 percent filter as above, but now restricts himself to cases where initial mispricing is less than 0.6 percent, the mean return is now 0.399 percent, or, at a reasonable minimum, 0.090–0.190 percent after transaction costs, which is still 4.8–10.4 percent when annualized.

An inspection of other filters and subperiods indicates that it is fair to say that the mean reversion tendencies of the mispricing component during the sample implies the existence of frequent speculative opportunities to index futures traders. It is notable that this exists in both the first and second subperiods. Thus one can infer that, several years into trading when the market exhibits arbitrage efficiency, there remain frequent cases of speculative inefficiency.[12]

IV. CONCLUSION

The main conclusion to be drawn is that it may be inappropriate for researchers to infer that index futures are now efficiently priced from the observation that arbitrage opportunities are infrequent. Though this is demonstrated using Canadian data, the insight can be extended to other markets. Given the higher liquidity and lower transaction costs associated with U.S. contracts (especially the *S&P 500*) it will not be surprising to find similar speculative profits available there. Further research is necessary to resolve this question.

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NOTES

1. The first Canadian index contract, which was based on the *Toronto Stock Exchange 300 Composite Index*, began trading in 1984 and reached its peak in 1986 with sales of nearly 60,000 contracts representing about 1.75 billion Canadian dollars. In 1987 this contract was phased out and replaced by the more narrowly based Toronto 35 contract. Consistent with U.S. experience, trading has not quite recovered to pre-Crash levels. The 300 contract had a multiplier of 10 for an average value over its life of roughly \$25,000–35,000; while the 35 has a multiplier of 500 for a dollar value per contract of roughly \$75,000–100,000 over the sample.

2. For one analytical derivation, see Modest and Sundaresan (1983).

3. Uncertainty plays a role as well, because theoretical futures prices are based on expected future dividends and interest rates, so what appears ex ante to be mispricing may not turn out to be ex post. Of course such risk declines as the contract approaches final settlement.

4. This is an approximation because it implicitly assumes no interest rate or dividend "surprises," as well as no revisions in expected interest rates and expected dividends (for the rest of the contract life). The effect of dividend uncertainty is slight if nearby contracts with a monthly cycle are used (as in the empirical exercise below), since stocks generally go ex-dividend several weeks after dividend announcements. If dividends are close to perfectly predictable, interest rate uncertainty is less problematic. The reason is that forward contracting can in this case be used to lock in reinvestment rates on the dividend payments.

5. This statement need not be true if dividends are large relative to the financing cost. Also, by revisions I mean both revisions in expectations beyond $t + T$, and unexpected realizations up to $t + T$. The latter will affect $F^*_{t,T+\tau}$, since this theoretical construct is based on information known only as of t.

6. The assumption that mispricing returns are uncorrelated with index returns seems reasonable a priori, but, in addition, I find that it cannot be rejected for any of the subperiods in my sample (or for the entire sample).

7. MacKinlay and Ramaswamy (1988) do, however, provide evidence of mispricing path-dependence (again using the *S&P 500*): contracts that have pierced either the upper or lower bound are shown to more likely again pierce the same boundary rather than the other one. They attribute this to the fact that arbitrage plays, initiated when the lower (upper) bound has been crossed, will generally be unwound prior to the upper (lower) bound being pierced.

8. In fact I find that in a regression of mispricing returns on initial mispricing levels, this independent variable has a (highly significant) negative coefficient, as one would expect.

9. Beginning at March 1985 seems appropriate since the evidence indicates that such contracts were usually priced in conformity with arbitrage efficiency by this point. Beyer (1985) and Deaves (1990) find frequent underpricing before March 1985. Nearby Wednesday *Toronto Stock Exchange* 300 futures settlement prices are used until June 17, 1987, virtually its last day of trading. The Toronto 35 contract is used from June 17, 1987 to the last Wednesday in the sample, April 19, 1989. The 300 contract initially used a quarterly contract cycle (like the *S&P 500*), but switched over in March 1985 to a monthly cycle (like the MMI contract). The 35 has always used a monthly cycle. For the full sample a "composite contract" is used. That is, the 300 is used until June 1987, and the 35 thereafter.

10. The relevant cash market index level is substituted for I_t. Since all contracts have a month or less before final cash settlement, the 30-day Government of Canada T-bill yield is deemed to be an appropriate very short-term risk-free rate. To calculate absolute dividend payments in index units for each future week up to the end of the contract (as well as for the last two day span from the final Wednesday to the Friday on which contracts are marked to market for the last time), the percentage

change in the 300/35 cash index over the week is subtracted from the percentage change in the corresponding 300/35 Total Return Indexes, and the difference is multiplied by I_t.

11. For the first subperiod mispricing sufficient to elicit arbitrage existed six percent of the time. Chamberlain, Cheung, and Kwan (1989) find mispricing existed about 13 percent of the time during November/1985–May/87. They use a 0.5 percent estimate (versus 0.6%) for transaction costs.

12. One possible criticism that can be leveled at these findings is countered by the similarity between the two subperiods. This is the criticism of non-synchronous trading, and it certainly applies to the *TSE 300* subperiod. Consider the following case of spurious profits. The market rises sharply at the close, which is immediately reflected in the future, but only partially reflected in the cash index because some stocks do not trade. As a result a "profit opportunity" emerges based on "mispricing." Of course there is really no profit to be garnered here, as it will be the cash index moving to the future, to eliminate mispricing, not the future moving to the cash. This criticism is certainly very much valid for the *TSE 300*, many of whose stocks trade infrequently, but it is not really valid for an index such as the Toronto 35, made up of many of the largest publicly-listed Canadian corporations whose shares trade frequently.

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